Princess Sumaya University for Technology
King Abdullah II School of Engineering

EE27355
Communication Principles

Quiz #2
Sunday 8/3/2026

Name:…………………………………………………………………… | | Section 1

Q.1) Evaluate the following integral:

$$\int_{-\infty}^{\infty} 5g(5-t)\delta(6-t)dt$$

<u>Solution:</u> [1-Point]

5g(-1)

Q.2) Evaluate the following:

$$\left(\frac{j\omega + 2}{\omega^2 + 9}\right)\delta(\omega)$$

<u>Solution:</u> [1-Point]

$$\left(\frac{2}{9}\right)\delta(\omega)$$

Q.3) If the energy of a signal g(t) is E_g, then use that to determine the energy of g(<u>*a*</u>t-*b*) if <u>*a*</u> is equal 0.5 and *b*=10.

Hint:

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2\, dt$$

<u>Solution:</u> [2-Points]

a=0.5….. Answer=$2E_g$

$$E_{g(at-b)} = \int_{-\infty}^{\infty} [g(at-b)]^2\, dt = \frac{1}{a}\int_{-\infty}^{\infty} g^2(\tau)\, d\tau = E_g/a$$

Q.4) In Figure Q.4, express signal $g_2(t)$ in terms of signal $g_1(t)$.

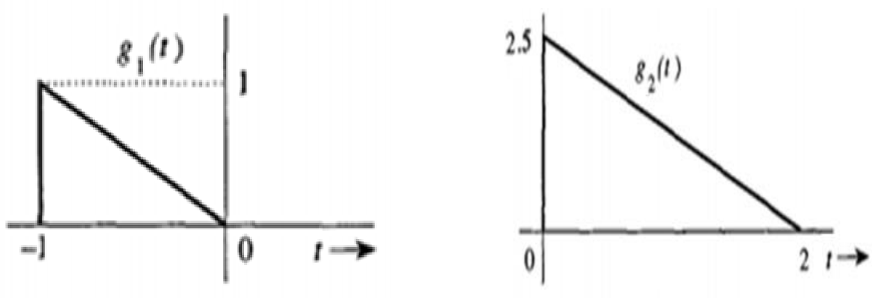


Figure Q.4

<u>**Solution:**</u> **[2-Points]**

$$g_3(t)=2.5g_1(0.5t-1)$$

Q.5) Find and sketch the Fourier Transform of the function shown in Figure Q.5 using the integral:

$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}\, dt \qquad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

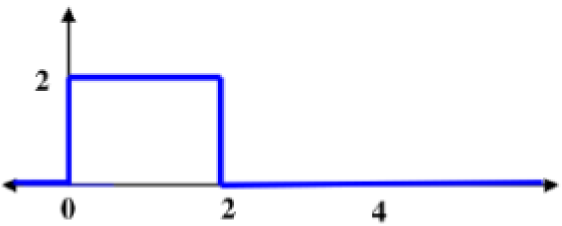


Figure Q.5

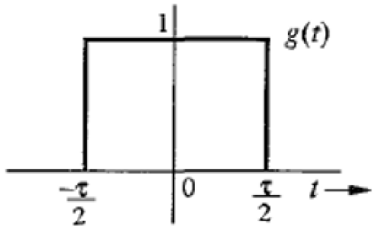
Hint:

Table 3.2
Fourier Transform Operations

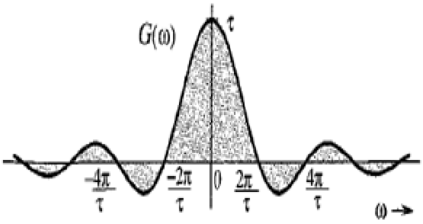
Operation	$g(t)$	$G(\omega)$
Addition	$g_1(t) + g_2(t)$	$G_1(\omega) + G_2(\omega)$
Scalar multiplication	$kg(t)$	$kG(\omega)$
Symmetry	$G(t)$	$2\pi g(-\omega)$
Scaling	$g(at)$	$\frac{1}{\lvert a\rvert} G\left(\frac{\omega}{a}\right)$
Time shift	$g(t - t_0)$	$G(\omega)e^{-j\omega t_0}$
Frequency shift	$g(t)e^{j\omega_0 t}$	$G(\omega - \omega_0)$
Time convolution	$g_1(t) * g_2(t)$	$G_1(\omega)G_2(\omega)$
Frequency convolution	$g_1(t)g_2(t)$	$\frac{1}{2\pi} G_1(\omega) * G_2(\omega)$
Time differentiation	$\frac{d^n g}{dt^n}$	$(j\omega)^n G(\omega)$
Time integration	$\int_{-\infty}^{t} g(x)\,dx$	$\frac{G(\omega)}{j\omega} + \pi G(0)\delta(\omega)$

Example 3.2

Find the Fourier transform of $g(t) = \text{rect}\,(t/\tau)$



$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}\,dt$$



$$G(\omega) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau}\right) e^{-j\omega t} dt$$

$$G(\omega) = \int_{-\tau/2}^{\tau/2} e^{-j\omega t} dt$$

$$= -\frac{1}{j\omega}\left(e^{-j\omega\tau/2} - e^{j\omega\tau/2}\right) = \frac{2\sin(\omega\tau/2)}{\omega}$$

$$= \tau \frac{\sin(\omega\tau/2)}{(\omega\tau/2)} = \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$$

$$\boxed{\text{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)}$$

$\text{sinc}\ (x) = 0$ when $x = \pm n\pi$.
$\text{sinc}\ (\omega\tau/2) = 0$ when $\omega\tau/2 = \pm n\pi$; that is, when $\omega = \pm 2n\pi/\tau$ $(n = 1, 2, 3, \cdots)$

Solution: [4-Points]

$$F\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi ft}\, dt = \int_{0}^{2} 2\, e^{-j2\pi ft}\, dt$$

$$= 2 \cdot \frac{e^{-j2\pi ft}}{-j2\pi f}\Big|_0^2 = 2 \cdot \frac{e^{-j4\pi f} - e^{0}}{-j2\pi f}$$

$$= 2 \left(\frac{1 - e^{-j4\pi f}}{j2\ \pi f} \right) = 2 \cdot e^{-j2\pi f} \left(\frac{e^{j2\pi f} - e^{-j2\pi f}}{j2 \cdot \pi f} \right)$$

$$= 2\ e^{-j2\pi f}\ \frac{\sin(2\pi f)}{\pi f} = 4\ e^{-j2\pi f}\ \frac{\sin(2\pi f)}{2\pi f}$$

$$= 4 e^{-j2\pi f} \operatorname{sinc}(2\pi f)$$

τ=2

Amplitude=2

Center is 1 so the shift is 1…. g(t-1)

Then the Fourier transform is G(ω)=(2)(2)sinc(ω)exp(-jω)= 4sinc(ω)exp(-jω)

$$g(t) = \text{rect}\left(\frac{t-1}{2}\right)$$

$$G(w) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t-4}{4}\right) e^{-jwt}\, dt$$

$$= \int_{0}^{2} 2\, e^{-jwt}\, dt$$

$$= \frac{2}{-jw}\, e^{-jwt}\Big]_{0}^{2}$$

$$= \frac{2}{-jw}\left[e^{-j2w} - e^{-j0w}\right]$$

$$= \frac{2}{-jw} e^{-jw}\left[e^{-jw} - e^{jw}\right]$$

$$= \frac{4}{w} e^{-jw}\left[\frac{e^{-jw} - e^{jw}}{2j}\right]$$

$$= \frac{4}{w} e^{-jw} \sin(w)$$

$$= 4 e^{-jw}\, \frac{\sin(w)}{w}$$

$$\boxed{G(w) = 4 e^{-jw}\, \text{sinc}(w)}$$

$w = \pi$